Application of the two-level response and parameter mapping for solution of inverse problem in Eddy Current Testing type-NDT

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Abstract — In this work, we develop two-level based methodology to solve the inverse problem of material structure recognition arising from Eddy Current Testing method. More precisely, we are interesting in identifying 3D surface crack inside a conducting, non-magnetic material. For the purpose of accelerating the time-consuming optimization of the forward model based on the numerical method, the reconstruction is provided by the minimization of a last-square functional using the Response and Parameter Mapping technique. In this way, the optimization burden was shifted from the time consuming and accurate model to a less exact but faster coarse surrogate. Here, the simulation in FEM was applied as a fine model, while the model based on the volume integral method (VIM) serves as a coarse model. This approach enables to shorten the evaluation time that is required to provide the proper parameter estimation of surface defects.

I. Introduction

The in-service inspection of steam generator tubing of pressurized water reactor type nuclear plant is recognized as a technical problem of a great importance for the nuclear power industry. One of the most popular NDT methods that can be applied for this purpose is the eddy current testing (ECT) technique. However, this non-destructive type of evaluation found also application in other fields of industry such as automotive, marine, aeronautic, and manufacturing, etc. for the inspection of critical structures. The main concept of this approach is based on testing an electrically conductive object with the time-varying electromagnetic field in order to measure response of ETC system. According to Faraday law, the information on the discontinuity is included in the measurement voltage or equivalently in the impedance of probe that is obtained for various frequencies of excitation current and different positions of the measurement sensor. Thus, the inverse problem relies on the recognition of defects parameters based on the signal from ETC measurements. In case of the direct optimization procedures such as Gauss-Newton algorithm or stochastic method, the computational time needed to reconstruct flaw parameter due to numerical model (FEM) may be very large. Therefore, the surrogate optimization based on the response and parameter mapping (RPM) technique [1] has been used for accelerating the time-consuming optimization problem. According to our knowledge, this efficient, engineering method was not until now applied for the purpose of material structure recognition based on the measurements data from the ECT system.

II. Space Mapping Methodology

Space mapping (SM) methodology, first proposed by Bandler et al in [1] recently, has become the subject of very intensive research in finding the solution of inverse problems in electromagnetism [2], [3]. Thus, its application in the context of 3D shape reconstruction from ECT data is promising.

Assuming the *j*-dimensional vector of the probe impedance in case of the coarse (fine) model for a certain *i*-dimensional defect parameters vector $\mathbf{x}_c \in X_c$ ($x_f \in X_f$) is denoted by $\mathbf{c}(\mathbf{x}_c) \in \Omega_c$ ($\mathbf{f}(\mathbf{x}_f) \in \Omega_f$), then the optimization problem is given by

$$\mathbf{x}_f^* = \arg\min_{\mathbf{x}_f \in X_f} \left\| \mathbf{f}\left(\mathbf{x}_f\right) - \mathbf{y} \right\|. \tag{1}$$

Here, $\mathbf{y}$ is the vector of the referenced impedance obtained by either simulation or as result of the conducted measurements. In the most popular form of SM method called an aggressive SM (ASM), the surrogate models in *k*-th iteration is defined as [1]

$$\mathbf{s}_{\mathrm{ASM}}^{(k)}\left(\mathbf{x}_f\right) = \mathbf{c}\left(\mathbf{p}^{(k)}\left(\mathbf{x}_f\right)\right), \tag{2}$$

with a mapping function is given by

$$\mathbf{p}^{(k)}\left(\mathbf{x}_f\right) = \mathbf{p}\left(\mathbf{x}_f^{(k)}\right) + \mathbf{B}^{(k)}\left(\mathbf{x}_f - \mathbf{x}_f^{(k)}\right), \tag{3}$$

where, $\mathbf{x}_f(k)$ is the *k*-th quasi Newton iteration with $\mathbf{B}(k)$ being an approximation of the Jacobian of $\mathbf{p}(\mathbf{x}_f)$.

In contrast to the ASM, the RPM algorithm applies the following surrogate model [5]

$$\mathbf{s}_{\mathrm{RPM}}\left(\mathbf{x}_f\right) = \mathbf{S}\left(\mathbf{c}\left(\mathbf{p}\ \left(\mathbf{x}_f\right)\right)\right), \tag{4}$$

with the response mapping function $\mathbf{S}$ and the parameter mapping function $\mathbf{p}$. When performing the affine mapping in an iterative way, the final form of an iterative surrogate model takes the form [4]

$$\begin{aligned}&\mathbf{S}^{(k)}\left(\mathbf{c}\left(\mathbf{p}^{(k)}\left(\mathbf{x}\right)\right)\right) = \\ &\mathbf{f}\left(\mathbf{x}^{(k)}\right) + \mathbf{D}^{(k)}\left(\mathbf{c}\left(\tilde{\mathbf{x}}^{(k)} + \mathbf{B}^{(k)}\left(\mathbf{x} - \mathbf{x}^{(k)}\right)\right) - \mathbf{c}\left(\tilde{\mathbf{x}}^{(k)}\right)\right),\end{aligned} \tag{5}$$

where the update of the rotation matrix $\mathbf{D}^{(k)}$ and $\mathbf{B}^{(k)}$, $\mathbf{x}^{(k)}$ is the recovered iterations scheme of the response mapping and the parameter mapping, while $\tilde{\mathbf{x}}^{(k)}$ is the iterations scheme obtained from the parameter mapping. In this way, the better convergence properties can be obtained. This is resulted from the use of the more general surrogate model, which can locally approximate in a better way the fine model. For a detailed description, see [4].

III. MODELS APPLIED IN TWO-LEVEL APPROACH IN ETC

Fig.1 presents a simplified configuration of the ECT system for the detection and evaluation of the 3D flaw located in the assumed search area.

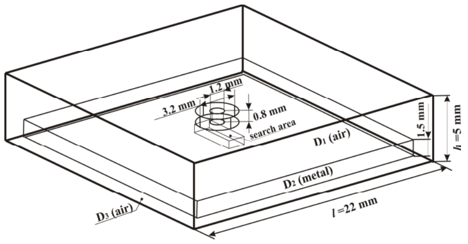

Fig. 1. The ECT system for flaw reconstruction.

A. *FEM simulation as a fine model*

In the ETC method 3D field distribution is governed by the Helmholtz equation in respect to vector magnetic potential **A** and scalar electric potential *V*. For this reason the COMSOL package has been used. During the construction of the 3D FEM model, the special attention was paid to the correct simulation of eddy current signals for given defects parameters.

B. *Reduced VIM approach as coarse model*

We derive the coarse model based on the reduced VIM approach under assumption that a good numerical approximation of electric fields can be reached by considering only *xx* component of the dyadic Green function [5]. As the consequence, the equivalent source of the perturbed field due to flaw is found by solving integral equations with the unknown as density dipoles multiplied by proper Green's function kernel [5]. The regularized Gauss-Newton algorithm [6] was used to find the rough solution of defects parameters.

IV. NUMERICAL RESULTS

For the purpose of validation of the proposed algorithm, we consider the reconstruction of an ellipsoidal crack shape with principal axes a = 0.3 mm, b = 1.0 mm, c = 0.5 mm, located on the same side as the probe. The analyzed model (Fig. 1.) is comprised of air domains D_1 and D_3 (ε0 , μ0) and the Inconel 600 plate: region D_2 (ε_0, μ_0, σ_0 = 0.98×106 S m^{-1}).The result of the simulation for 49 positions of the asymmetrically placed pancake coil: x_{c0} = 1mm, y_{c0} = 0 mm excited by time-harmonic current at f = 100 kHz is shown in Fig. 2. The result of optimization by means of ASM algorithm was summarized in Table 1, where the last column represents the mean relative error. This error value is caused mainly by the misalignment between both model responses. Further elaboration of methodology shall allow to compensate it and this will be the subject of our forthcoming work.

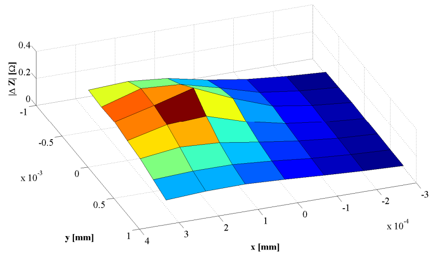

Fig. 2. The variation of probe impedance for ellipsoidal crack.

TABLE I
THE COURSE OF ASM OPTIMIZATION OF ANALYZED DEFECTS [7].

Name of defect	Initial point	Reconstructed size of defect	MRE for $\hat{\mathbf{x}}$
Crack (*ellipsoidal flaw*)	$\begin{bmatrix} 0.2 \\ 0.75 \\ 0.375 \end{bmatrix}$	$\begin{bmatrix} 0.084 \\ 0.978 \\ 0.546 \end{bmatrix}$	27%

V. CONCLUSION

The application of the two-level approach to 3D flaw identification enables to reduce the computational complexity of defects identification procedures in comparison with other fine model optimization methods e.g. the regularized Gauss-Newton method or stochastic algorithms. However, the inversion procedure based on the ASM techniques might fail in case of the significant misalignment between both models responses. In such situation, there is a need to implement other space mapping technique like e.g. the RPM approach in order to improve the accuracy of defects reconstruction.

VI. REFERENCES

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